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Exclusive photoproduction of Φ on proton in the quark–diquark model

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Abstract. We present predictions for exclusive photoproduction of a Φ meson on proton at large transfer, where we use a quark–diquark structure model for the proton. Extrapolation from our results to lower transfers is comparable in magnitude with available data in that range. This may support the diquark model in its ability to provide, for that process, an appropriate link between diffractive physics at low transfer and the standard semiperturbative approach of hard exclusive processes at very large transfer, in which the proton recovers its three-quark structure.

1 Introduction

Up to now, exclusive photoproduction of a vector meson V ($V = \rho, \omega, \Phi, J/\Psi$) from a proton,

$$\gamma + p \to V + p \tag{1}$$

has been measured mainly at very low values of t, t being the opposite of the squared momentum transfer at the proton vertex. In that region (e.g., $t \leq 1$ GeV^2), and in a wide energy range up to HERA energies, the observed characteristics of photoproduction of the lighter mesons are those of a soft diffractive process. As has been shown by Donnachie and Landshoff [1], this can be well described for the most part in a picture using both the vector dominance model (VDM) and Pomeron phenomenology: there, the incoming photon is assumed to convert into a vector meson which afterwards exchanges a soft Pomeron with the proton target. In this respect, one may consider reaction (1) in this small-transfer range as a good testing bench for Pomeron physics. Indeed, that picture works nicely in the case of photoproduction of lighter mesons ρ, Φ, and ω [2]. However, it fails to reproduce the energy dependence of the cross section for J/Ψ photoproduction [3].

In a QCD-inspired picture, Pomeron exchange is commonly modeled as the effective exchange of two nonperturbative gluons. Donnachie and Landshoff substantially improved that picture [4]. When applied to photoproduction of light mesons, their two-gluon exchange model leads to results very similar to those provided by the Pomeron-exchange model of the same authors [5]. However, the model fails to describe the energy dependence of J/Ψ photoproduction, and that of virtual photoproductions of ρ, Φ and J/Ψ, as has been observed at HERA [6].

Following the works of Ryskin and of Brodsky, et al. [7], this may reveal that QCD perturbative effects enter the game, since a large momentum scale (either the mass of the heavy vector meson produced in the case of J/Ψ, or the high Q^2 of the virtual photon in the case of ρ) appears in the reaction. In their approach, Brodsky, et al. wrote the amplitude of the process as the product of three terms: an amplitude describing the breaking of the virtual photon into a $q\bar{q}$ pair, the valence $q\bar{q}$ wave function of the vector meson and an amplitude describing a nonperturbative two-gluon structure of the proton. Actually, the latter amplitude plays a crucial role in this approach, since the dependence on energy of the cross section directly reflects the small-x behavior of the gluon momentum distribution in the proton. In this way, one can account for the rapid rise with energy of the cross sections that has been observed at HERA.

In this paper, we consider (real) photoproduction of Φ on proton at larger t. Since the Φ meson is a pure $s\bar{s}$ state and the strangeness content of the nucleon wave function is probably small, that process is dominated at lowest order in QCD by the two-gluon exchange mechanism and thus provides a unique way to study the latter. This process has already been investigated at moderate t by Laget and Mendez–Galain [8], who use the nonperturbative picture of Donnachie and Landshoff. Their prediction for the differential cross section $\mathrm{d}\sigma/\mathrm{d}t$ at infinite photon energy exhibits a node at $t = 2.4$ GeV^2, which could in turn serve as a test of the model. Unfortunately, the only existing data for Φ production with real photons or virtual photons correspond to very low values of t.

Since higher values of t provide larger momentum scales, it is tempting to apply in that range the semiperturbative approach of hard exclusive processes developed long ago by Brodsky, Farrar, and Lepage (BFL) [9], and by Chernyak and Zhitnitsky (CZ) [10]. This approach has been described and discussed many times in the literature and been shown to provide the correct order of magnitude for numerous exclusive amplitudes. Indeed, Farrar, et al. already applied it to various photoproduction processes [11].

We recall here that in this formalism, the amplitude of a given exclusive process is obtained by a convolution formula of relativistic hadron wave functions with elementary hard-scattering amplitudes that involves the valence quarks and antiquarks of the hadrons taking part in the reaction. When using the baryon wave functions derived from QCD sum rules by CZ, this method provides a correct order of magnitude for many exclusive amplitudes involving baryons. However, there exist important subasymptotic helicity-flip effects that do not fit the above picture, where any spin effect is to be described by the so-called helicity conservation rule [12]. While this rule should be valid asymptotically, it appears to be inconsistent with most experimental data at intermediate energies. To cure this failure for processes involving baryons, a quark–diquark structure of baryons has been proposed [13]. In that alternative picture, two of the three quarks of a baryon cluster together in a diquark structure. In the subasymptotic region, diquarks are supposed to act as quasi-elementary constituents having direct couplings with photons and gluons. Helicity flips are then caused by vector diquarks. On the other hand, diquarks should asymptotically dissolve into quarks, restoring the usual three-quark picture of baryons. The diquark hypothesis provides natural explanations for many phenomena that are otherwise difficult to describe by standard models [14]. In the following, we apply that picture, as a first semiperturbative calculation of the process under study, in the framework of the BFL scheme, from moderate to large values of t.

Another point of theoretical interest in the study of elastic Φ photoproduction is found in the structure of the amplitude of the underlying hard-scattering process. Indeed, this amplitude exhibits singularities coming from on-shell quark lines. Farrar, et al. [15] have shown that in fact, any exclusive photoproduction process is, to leading twist, insensitive to long-distance physics and does not require Sudakov resummation. The propagator singularities are integrable, and their presence does not affect the validity of the hard scattering approach. The appearance of an imaginary part of the amplitude at leading order in α_s is thus considered as a nontrivial prediction of perturbative QCD.

Studies of J/Ψ or η_c photoproduction are of course of the same interest as that of Φ photoproduction, since the charm content of the proton is probably negligible too, or even nonexistent. However, on one hand, the high value of the c-quark mass is a source of computational complications, and, on the other hand, more complicated graphs are involved in case of η_c production. Thus we leave these two processes for future investigations.

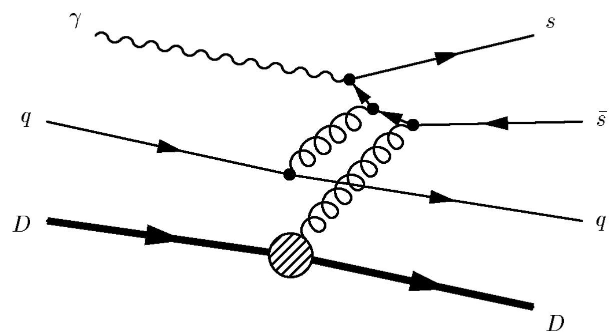

Fig. 1. A typical diagram for $\gamma + p \to \Phi + p$ in the quark–diquark picture

In Sect. 2, a short description of the quark–diquark model for exclusive processes involving the proton is presented. The details of calculation of the hard-scattering amplitude for Φ photoproduction are given in Sect. 3. In Sect. 4, we give our numerical results and concluding remarks.

2 The quark–diquark model

Let us first notice that formally, the amplitudes of the two processes $\gamma + p \to V + p$ and $V \to p + \bar{p} + \gamma$ are related just by crossing. Two of us have already studied the decay process $J/\Psi \to p + \bar{p} + \gamma$, using the quark–diquark model structure of the proton. Thus we refer largely to our previous paper [16] for notations.

The formalism we use below is the same as that of BFL, except that the three-body structure of the proton is replaced by a two-body one. To lowest order in QCD, the photoproduction of Φ on proton is thus described by the generic graph of Fig. 1. The corresponding amplitude is obtained here too from a convolution formula

$$T = K \int [\mathrm{d}x][\mathrm{d}x'][\mathrm{d}y] \frac{\alpha_{\mathrm{s}}^2}{g^2 G^2} T_{\mu\nu\alpha\sigma} \varepsilon^{*\alpha}_{(\phi)} \varepsilon^{\sigma}_{(\gamma)} I^{\mu\nu} \qquad (2)$$

where $[\mathrm{d}x] = \delta(1 - x_1 - x_2)\mathrm{d}x_1\mathrm{d}x_2$, $[\mathrm{d}x'] = \delta(1 - x'_1 - x'_2)\mathrm{d}x'_1\mathrm{d}x'_2$, $[\mathrm{d}y] = \delta(1 - y_1 - y_2)\mathrm{d}y_1\mathrm{d}y_2$. As is currently done in the standard BFL model, all constituent transverse momenta are here neglected in the hard-scattering amplitude. In other words, collinearity of the constituents with the parent hadron is assumed: $x_1 = x$ ($x'_1 = x'$) is the four-momentum fraction of the quark inside the ingoing (outgoing) proton, and $x_2 = 1 - x$ ($x'_2 = 1 - x'$) that of the accompanying diquark; $y_1 = y$ ($y_2 = 1 - y$) is the four-momentum fraction of the strange quark (that of the strange antiquark) inside the Φ meson. The tensor $T_{\mu\nu\alpha\sigma}$ is the amplitude for the subprocess $gg\gamma \to \Phi$; the two space-like gluons have four-momenta $g = xp - x'p'$ and $G = (1 - x)p - (1 - x')p'$, respectively, p being the four-momentum of the ingoing proton and p' that of the outgoing proton. $\varepsilon^{\alpha}_{(\phi)}$ and $\varepsilon^{\sigma}_{(\gamma)}$ are polarization vectors for the Φ

and for the photon respectively, $I^{\mu\nu}$ is a tensor amplitude describing the two-gluon scattering by a quark–diquark system, and K is the overall normalization factor:

$$K = \sqrt{4\pi\alpha} e_{\mathbf{s}} \frac{4\pi^2 f_\phi}{9\sqrt{6}} C \quad (3)$$

with the color factor $C = -2/(3\sqrt{3})$ and the Φ decay constant $f_\phi \sim 150$ MeV.

For the sake of consistency of the model, we have neglected the masses of the constituents, as well as those of the parent hadrons, whenever possible. This led us to use the relativistic form of the Φ wave function. Depending on the helicity h of the Φ, it is given by

$$\Psi_\phi = \frac{f_\phi}{\sqrt{24}} \frac{1}{\sqrt{3}} \sum_{\text{color}} s\bar{s} \begin{cases} \not{P}\ \phi_{\rm L}(y) & \text{for} \quad h = 0 \\ \not{P}\ \not{\epsilon}^{(h)}_{(\phi)}\ \phi_{\rm T}(y) & \text{for} \quad h = \pm 1 \end{cases} \quad (4)$$

where $\phi_{\rm L}(y)$ and $\phi_{\rm T}(y)$ are normalized y distributions for, respectively, a longitudinally and a transversally polarized meson.

In that approximation, and because of the particular structure of the amplitude of the subprocess (an odd number of γ matrices), it appears then that only longitudinal Φ are produced.

The color factor is the same for all diagrams. This is why the amplitude for $\gamma gg \to s\bar{s}$ is formally the same as the amplitude for 3γ's $\to e^+e^-$, leaving aside the color factor and coupling constant. We can thus obtain the tensor amplitude $T_{\dots}$ in (2) from the tensor amplitude of $3\gamma \to e^+e^-$ with massless electrons by removing the coupling constant, using appropriate four-momenta, and making the substitution

$$V_{e^+} \bar{U}_{e^-} \to \not{P}. \quad (5)$$

The wave functions of mesons have been derived by CZ from QCD sum rule technics. Here we use their longitudinal Φ wave function

$$\phi_{\rm L}(y) = 6y(1-y)\left\{y(1-y) + 0.8\right\} \quad (6)$$

that can be found in [10], p. 259.

Allowing for both scalar (S) and vector (V) diquarks, a quark–diquark proton state corresponding to an up or down proton helicity takes on the general form:

$$\begin{aligned} |p^{\uparrow\downarrow}> \ \sim \ & - f_{\mathbf{s}}\ [\ 2\phi_1(x) + \phi_3(x)\ ]\ S(ud)\ u^{\uparrow\downarrow} \pm \\ & f_{\mathbf{v}}\ [\ \phi_2(x)\left\{\sqrt{2}\ V_\pm(ud)\ u^{\downarrow\uparrow}\ -2\ V_\pm(uu)\ d^{\downarrow\uparrow}\right\} \\ & +\phi_3(x)\left\{\sqrt{2}\ V_0(uu)\ d^{\uparrow\downarrow} - V_0(ud)\ u^{\uparrow\downarrow}\right\}\ ] \end{aligned} \quad (7)$$

where $V_h(q_1q_2)$ is an isovector–(pseudo)vector diquark state made of two quarks having flavors q_1 and q_2 and $h = 0, \pm 1$ is its helicity; $S(ud)$ is the isoscalar–scalar diquark state.

The $\phi_i(x)$ are normalized wave functions, and $f_{\mathbf{s}}$ and $f_{\mathbf{v}}$ are normalization constants that may be chosen as unequal to allow for various admixtures of scalar and vector components. Expressions of diquark–gluon couplings have been given in [17]. Using obvious notations and omitting color factors as well as coupling constants, we find these expressions, in the space-like channel ($g + D \to D'$), to be

$$(S'\ S)_\mu = F_{\mathbf{s}}\ (D_\mu + D'_\mu) \quad (8)$$

for a pair of scalar diquarks, and

$$(V'_{h'} V_h)_\mu = -F_1\ (D_\mu + D'_\mu)\varepsilon^{h'\,*}_{D'}.\varepsilon^h_D + F_2\ \left\{(D.\varepsilon^{h'\,*}_{D'})\varepsilon^h_{D\mu} + (D'.\varepsilon^h_D)\varepsilon^{h'\,*}_{D'\mu}\right\} - F_3\ (\varepsilon^{h'\,*}_{D'}.D)(\varepsilon^h_D.D')(D_\mu + D'_\mu) \quad (9)$$

for a pair of vector diquarks[1].

The F above are the diquark form factors depending on $Q^2 = -G^2 = -(D - D')^2$. A possible parametrization, aimed at describing the natural evolution of the diquark model into the usual three-quark picture, has been proposed by the authors of [17]. It has the following form:

$$F_{\mathbf{s}}(Q^2) = \chi \frac{Q_0^2}{Q^2 + Q_0^2}\ ,\ F_1(Q^2) = \chi\left(\frac{Q_1^2}{Q^2 + Q_1^2}\right)^2$$
$$F_2(Q^2) = (1 + k_{\mathbf{v}})F'_1(Q^2)\ ,\ F_3(Q^2) = \frac{Q^2 F_1(Q^2)}{(Q^2 + Q_1^2)^2}, \quad (10)$$

with

$$\chi = \begin{cases} \alpha_{\mathbf{s}}(Q^2)/\alpha_{\mathbf{s}}(Q_0^2) & \text{for} \quad Q^2 \geq Q_0^2 \\ 1 \quad \text{for} \quad Q^2 \leq Q_0^2, \end{cases} \quad (11)$$

$k_{\mathbf{v}}$ being the anomalous magnetic moment of the vector diquark. The value $k_{\mathbf{v}}$=1 is commonly assumed. The above-defined evolutionary picture also induces one to use, for the sake of consistency, coupling constants of the running form and to set $\alpha_{\mathbf{s}}^2 = \alpha_{\mathbf{s}}(-g^2)\alpha_{\mathbf{s}}(-G^2)$, with $-g^2 = txx'$, but to restrict $\alpha_{\mathbf{s}}$ to some maximum value c_1. Recall that setting the factor χ in diquark form factors provides the correct power of α_s in amplitudes at large transfer.

In [16], we used such a parametrization to fit the proton magnetic form factor G_M in the space-like region. Modeling the momentum fraction distributions by a wave function of asymptotic form, i.e., taking

$$\phi_1(x) = \phi_2(x) = \phi_3(x) = \phi_{\rm as}(x) = 20x(1-x)^3, \quad (12)$$

we obtained a quite good fit with the following parameter values:

$$f_{\mathbf{s}} = 40 \text{ MeV}\ ,\ f_{\mathbf{v}} = 96 \text{ MeV}\ ,\ Q_1^2 = 2 \text{ GeV}^2\ ,$$

$$Q_0^2 = 2.3 \text{ GeV}^2$$

[1] We do not consider here a possible mixed coupling involving both scalar and vector diquarks, as it is commonly expected to give a small contribution.

$$\text{for } c_1 = 0.3 \tag{13}$$

where, as said above, c_1 is the maximum allowed value of the running coupling constant $\alpha_{\mathbf{s}}$. Given that success, we used the same parametrization for the present calculation.

3 The hard-scattering subamplitudes

As already mentioned, according to the model here used, longitudinal Φ are preferentially produced. There are then *a priori* eight dominant helicity amplitudes describing the process, which we denote by $T^{\lambda\lambda' \Lambda}$, in which λ λ' and Λ are the helicities of, respectively, the ingoing proton, the outgoing proton, and the real incident photon. Thanks to parity and rotational invariance, that number in fact reduces to four, and we have:

$$\begin{array}{ll} T^{\downarrow\downarrow -} = -T^{\uparrow\uparrow +} & T^{\downarrow\downarrow +} = -T^{\uparrow\uparrow -} \\ T^{\downarrow\uparrow -} = T^{\uparrow\downarrow +} & T^{\downarrow\uparrow +} = T^{\uparrow\downarrow -} \end{array} \tag{14}$$

To be more specific, let us now concentrate on the calculation of the amplitude $T^{\uparrow\uparrow +}$. To compute the amplitudes, we have chosen for convenience the polarization states of the particles according to a "t-channel helicity-coupling scheme" [18]. In that scheme, the photon helicities represent photon spin projections on the "vertex plane" defined by the Φ and photon four-momenta, and the photon polarization vectors $\varepsilon^{(\pm)}_{(\gamma)}$ are then perpendicular to that plane. Similarly, the helicities of the protons are projections of their respective spins on the vertex plane defined by their two four-momenta.

From (2), we thus obtain

$$T^{\uparrow\uparrow +} = K \frac{\sqrt{2t}}{t^2} \int [\mathrm{d}x][\mathrm{d}x'][\mathrm{d}y] \frac{\alpha_{\mathbf{s}}^2}{x(1-x)x'(1-x')} \phi_{\mathrm{L}}(y) \mathcal{T}^{\uparrow\uparrow +}, \tag{15}$$

where

$$\mathcal{T}^{\uparrow\uparrow +} = 12\sqrt{s}\frac{f_v^2}{\mu_v} F_2(Q^2)(1-x)(1-x')\cot(\theta/2) \times \frac{1}{y(1-y)} \left\{ \frac{(1-y)A}{d'} + \frac{yB}{d} - \frac{y(1-y)C\sin^2(\theta/2)}{dd'} \right\}, \tag{16}$$

s being the center-of-mass energy squared, θ the Φ emission angle relative to the incident photon direction in the center-of-mass frame ($\sin^2(\theta/2) = t/s$), $Q^2 \sim t(1-x)(1-x')$, and μ_v the diquark mass, usually taken to be equal to 600 MeV; d and d' are the s-quark propagator factors

$$\begin{aligned} d &= xx'\sin^2(\theta/2) + y(x\cos^2(\theta/2) - x') - \mathrm{i}\epsilon, \\ d' &= (1-x)(1-x')\sin^2(\theta/2) + \\ &(1-y)((1-x)\cos^2(\theta/2) - (1-x')) - \mathrm{i}\epsilon, \end{aligned} \tag{17}$$

for which, following the usual prescription, $\epsilon \to 0^+$. Finally, A, B, and C are given by

$$\begin{aligned} A &= \phi_3(x')\phi_2(x)((1-x')\sin^2(\theta/2) + (1-y)\cos^2(\theta/2)) \\ &\quad -\phi_2(x')\phi_3(x)((1-x)\sin^2(\theta/2) - (1-y)), \\ B &= \phi_2(x')\phi_3(x)(x'\sin^2(\theta/2) + y\cos^2(\theta/2)) \\ &\quad -\phi_3(x')\phi_2(x)(x\sin^2(\theta/2) - y), \\ C &= \phi_2(x')\phi_3(x)[(y(1-x) - x(1-x))\cos^2(\theta/2) + \\ &\quad x'(1-y) - x'(1-x')] + \phi_3(x')\phi_2(x)[(x(1-y) - \\ &\quad x(1-x))\cos^2(\theta/2) + y(1-x') - x'(1-x')]. \end{aligned} \tag{18}$$

From (17), it is clear that the kernel (15) has singularities within the domain of integration, since the real parts of d and d' have zeros in x located respectively at

$$z_0 = \frac{x'y}{y\cos^2(\theta/2) + x'\sin^2(\theta/2)} \leq 1 \quad \text{and} \quad z_1 = 1 - \frac{(1-x')(1-y)}{(1-y)\cos^2(\theta/2) + (1-x')\sin^2(\theta/2)} \leq 1. \tag{19}$$

These singularities correspond to one or the two exchanged s-quarks going on-shell in the graph of Fig. 1. It is important to notice that when $x' = y$, the two zeros coincide and are both equal to x' (or y). The one-pole terms ($\sim 1/d$ or $\sim 1/d'$) can be treated readily by use of the general formula

$$\frac{1}{u - \mathrm{i}\epsilon} = \mathcal{P}\left(\frac{1}{u}\right) + \mathrm{i}\pi\delta(u), \tag{20}$$

where $\mathcal{P}$ denotes the principal value. The two-pole term $\sim 1/(dd')$ corresponds to the graph where the photon line is sandwiched between the two gluon lines. Setting $r = \Re(d), r' = \Re(d')$ ($\Re$ means the real part), one gets

$$\frac{1}{dd'} = \mathcal{P}\left(\frac{1}{r}\right)\mathcal{P}\left(\frac{1}{r'}\right) - \pi^2\delta(r)\delta(r') + \mathrm{i}\pi\left\{\mathcal{P}\left(\frac{1}{r}\right)\delta(r') + \mathcal{P}\left(\frac{1}{r'}\right)\delta(r)\right\}. \tag{21}$$

It appears that the product of two delta functions, which is apparently the most singular term, leads in fact to a null contribution. This is attributable to the fact that the amplitude of a fermion–antifermion–vector meson vertex is zero when all particles are massless.

Let us first consider the imaginary part of the full amplitude. It has the general form

$$C_1(x,x',y)\delta(r) + C_2(x,x',y)\delta(r') + C_3(x,x',y)\left\{\mathcal{P}\left(\frac{1}{r}\right)\delta(r') + \mathcal{P}\left(\frac{1}{r'}\right)\delta(r)\right\}, \tag{22}$$

which can be trivially integrated by hand over the variable x, yielding an expression of the form:

$$\frac{C_1(z_0,x',y)}{\alpha}+\frac{C_2(z_1,x',y)}{\alpha'}+ \frac{1}{\alpha'}C_3(z_1,x',y)\mathcal{P}\left(\frac{1}{r}\right)_{x=z_1}+\frac{1}{\alpha}C_3(z_0,x',y)\mathcal{P}\left(\frac{1}{r'}\right)_{x=z_0}, \tag{23}$$

in which $\alpha = x'\sin^2(\theta/2)+y\cos^2(\theta/2)$ and $\alpha' = (1-x')\sin^2(\theta/2)+(1-y)\cos^2(\theta/2)$. One must be cautious with the last two terms, for they are a source of difficulties in the subsequent (numerical) integrations, as we now explain. Since

$$\frac{1}{\alpha'}\mathcal{P}\left(\frac{1}{r}\right)_{x=z_1}=\frac{1}{\alpha}\mathcal{P}\left(\frac{1}{r'}\right)_{x=z_0}=\frac{4}{\sin^2\theta}\mathcal{P}\left(\frac{1}{(x'-y)^2}\right), \tag{24}$$

a double-pole-like term $\sim 1/(x'-y)^2$ appears. However, that "singularity" is tempered by zeros of the factors C_3 when $x'=y$ ($C_3 \propto (x'-y)$). These zeros, which are of degree 1, are reminiscent of the already mentioned fact that the amplitude of a fermion–antifermion–vector meson vertex is zero when all particles are massless; and, precisely, the two exchanged s-quarks that are coupled to the real photon in the corresponding "singular" Feynman graph are both massless when $x=x'=y$. Consequently, the (seemingly) double pole reduces to a simple pole:

$$(x'-y)\ \mathcal{P}\left(\frac{1}{(x'-y)^2}\right)\to\mathcal{P}\left(\frac{1}{x'-y}\right). \tag{25}$$

In order to manage this in a cautious way, we proceed as follows. First, we split the products of wave functions ϕ into symmetrical and antisymmetrical parts:

$$\begin{aligned}S_{23}(x,x')&=\frac{1}{2}\left\{\phi_2(x')\phi_3(x)+\phi_3(x')\phi_2(x)\right\},\\ A_{23}(x,x')&=\frac{1}{2}\left\{\phi_2(x')\phi_3(x)-\phi_3(x')\phi_2(x)\right\}.\end{aligned} \tag{26}$$

Of course, this operation is useful only when $\phi_2 \neq \phi_3$ and is thus inoperant for the symmetrical parametrization used in this paper. We present it here just for further applications. The imaginary part of the factor in brackets in formula (16) may then be rewritten as

$$\begin{aligned}&\pi\delta(x-z_0)\left\{S_{23}(z_0,x')\mathcal{S}_0+A_{23}(z_0,x')\mathcal{A}_0\right\}+\\ &\pi\delta(x-z_1)\left\{S_{23}(z_1,x')\mathcal{S}_1+A_{23}(z_1,x')\mathcal{A}_1\right\}.\end{aligned} \tag{27}$$

In order to maximally reduce the effect of the pseudopole $\propto 1/(x'-y)$, we further make the shift $x'=y+(x'-y)$ and the appropriate simplifications in all coefficients $\mathcal{S}$ and $\mathcal{A}$. We then arrive at the more manageable expressions:

$$\begin{aligned}\mathcal{S}_0=\frac{y}{\alpha^2}&\left\{x'(x'-y)\sin^4(\theta/2)+y\alpha(1+\cos^2(\theta/2))+\right.\\ &(1-y)\left(2x'(2x'-1)+\cos^2(\theta/2)\left[(y-x')(2x'-1)\right.\right.\\ &\left.\left.-2y^2+y\sin^2(\theta/2)\right]\right)-\frac{(1-y)}{\cos^2(\theta/2)}\left[(x'+y)(2x'-1)\right.\\ &\left.\left.+2y^2\right]+y^2(1-y)(1-2y)\frac{(1+\cos^4(\theta/2))}{\cos^2(\theta/2)}\mathcal{P}\left(\frac{1}{x'-y}\right)\right\},\end{aligned} \tag{28}$$

$$\begin{aligned}\mathcal{A}_0=\sin^2(\theta/2)\frac{y}{\alpha^2}&\left\{x'y+\sin^2(\theta/2)(x'-y)^2+x'-y+\right.\\ &y(1-y)\cos^2(\theta/2)-\frac{1}{\cos^2(\theta/2)}(1-y)(x'+y)\\ &\left.-y^2(1-y)\frac{1+\cos^2(\theta/2)}{\cos^2(\theta/2)}\mathcal{P}\left(\frac{1}{x'-y}\right)\right\}.\end{aligned} \tag{29}$$

The other factors $\mathcal{S}_1$ and $\mathcal{A}_1$ are obtained from $\mathcal{S}_0$ and $-\mathcal{A}_0$, respectively, by the simple replacement $x'\to 1-x'$, $y\to 1-y$.

Let us now turn to the real part of the amplitude. It appears that different terms of the same (large) magnitude compensate for each other in the domain of integration. To cure that new difficulty which causes numerical uncertainties, we decided to put all the expressions on the same denominator rr' and to again introduce symmetrical and antisymmetrical combinations of wave functions so that the real part of the factor in the brackets in (16) takes on the form

$$\left\{S_{23}(x,x')\mathcal{S}'+A_{23}(x,x')\mathcal{A}'\right\}\mathcal{P}\left(\frac{1}{r}\right)\mathcal{P}\left(\frac{1}{r'}\right). \tag{30}$$

Then, we set $u=x'-x$, $v=y-x'$, and rewrite the coefficients $\mathcal{S}'$ and $\mathcal{A}'$ as polynomials in u and v. We thus get

$$\mathcal{S}'=v^3H_3+v^2H_2+vH_1+H_0, \tag{31}$$

with

$$\begin{aligned}H_3&=-4u\cos^4(\theta/2)+2\sin^2(\theta/2)(1+\cos^2(\theta/2))\\ &\quad\times(1-2x'),\\ H_2&=2u(1-2x')(1+2\cos^4(\theta/2))+\\ &\quad\sin^2(\theta/2)(1+\cos^2(\theta/2))(8x'(1-x')-1),\\ H_1&=u^2\sin^4(\theta/2)(1-2x')+2u\sin^4(\theta/2)x'(1-x')\\ &\quad+2u\sin^2(\theta/2)(1-4x'(1-x'))+2u(6x'(1-x')-1)\\ &\quad-2\sin^2(\theta/2)(1+\cos^2(\theta/2))x'(1-x')(1-2x'),\\ H_0&=ux'(1-x')\left\{(2u-2x'+1)\sin^4(\theta/2)\right.\\ &\quad\left.-2(1-2x')\right\},\end{aligned} \tag{32}$$

and

$$\mathcal{A}' = \sin^4(\theta/2)\left\{-2v^3 + v^2(1-2u-2x') + v(2x'(1-x') - u^2) + ux'(1-x')\right\}. \quad (33)$$

This trick is supposed to moderate both the above-mentioned cancellations and the double pole $\sim 1/(rr')$. At this point, notice that when $v \to 0$, one has $1/(rr') \sim 1/(u^2x'(1-x'))$, whereas the numerators of the amplitudes behave like $ux'(1-x')$, so that the double pole in u turns into a simple pole (with no end-point singularities). Similarly, when $u \to 0$, then $1/(rr') \sim 1/(v^2x'(1-x'))$, while the numerators of the amplitudes are $\sim vx'(1-x')$, so that the amplitude is only $\sim 1/v$.

As for end-point singularities due to the denominator $x(1-x)x'(1-x')y(1-y)$ coming from gluon and s-quark propagators, they cause no problem, since the above denominator is canceled by an equivalent factor contained in the product of hadron wave functions. We assume that this cancellation of end-point singularities is sufficient, i.e., that no extra Sudakov form factor that could *a priori* suppress those singularities more drastically is to be implemented. The problem of possible Sudakov suppression of end-point singularities is beyond the scope of the present work. It has been discussed in different contexts by various authors, to whom we refer the reader [19]. For our calculations, given the uncertainties in the parametrization of the diquark model in its present form, we think it senseless to introduce an additional complication with a Sudakov factor, the precise form of which, for that model and for the process here studied, is yet unknown anyway. Nevertheless, we apply here a kind of suppression of end-point singularities by cutting off the dangerous growth of coupling constants $\alpha_s(-g^2)$ and $\alpha_s(-G^2)$ in the end-point region by means of the parameter c_1 (see above).

All amplitudes have been treated in the same way. Moreover, to prevent additional instabilities in our numerical evaluations, we have modeled principal values by the approximate form

$$\mathcal{P}\left(\frac{1}{z}\right) \sim \frac{z}{\epsilon^2 + z^2} \quad (34)$$

with $\epsilon \ll 1$. For instance, in real parts, we made the substitution

$$\mathcal{P}\left(\frac{1}{rr'}\right) \sim \frac{r}{\epsilon_1^2 + r^2}\frac{r'}{\epsilon_2^2 + r'^2} \quad (35)$$

Our numerical results are presented in the next section.

4 Results and conclusions

We have spent a lot of time in searching for the best method of integration and checking the stability of our computations. Given the rather simple dependence on y of the integrant, we tried to integrate, by hand, over y first, following a method very similar to the one used in [21]. However, this is not a good method in the present computation, because it induces spurious singularities in the subsequent integration over x and x' through denominators such as $x\cos^2(\theta/2) - x'$, which does have a zero in the integration domain. Applied to an integration over x', the method has the same drawback. We also tried a numerical integration in the complex y plane, but this too led to intractable instabilities. Thus, we were forced to integrate first over x. As was said in Sect. 3, different terms of the same magnitude compensate for each other in the domain of integration, and this effect is increased by the presence of double-pole factors. The method used in [21] which implies additional subtractions does not cure that drawback. This is why we finally adopted the strategy described in Sect. 3. We then varied the parameters ϵ down to a value as small as 10^{-9} and even cross-checked our results using two different programs of integration, namely a Gaussian quadrature method (RGAUSS) and a Monte Carlo method (VEGAS). It appears that the real parts of amplitudes are the most intractable: the lower the value of ϵ, the greater should be the number of calls of the integrant. However, a value of ϵ between 10^{-4} and 10^{-5} seems to provide the best stability. By chance, for the particular parametrization here used, in the best part of the kinematical range we have investigated, the contributions of real parts appear to be much less than those of imaginary parts, by a factor of a few percent or less. On the other hand, the results obtained for the imaginary parts are very stable, being much less sensitive to the value of ϵ. Thus, the results we are presenting now account for the imaginary parts of amplitudes only.

In Fig. 2, is shown the momentum transfer distribution we obtain of the proton–photon invariant mass W in the range 2 GeV^2–10 GeV^2 and for two values, 5 GeV (CEBAF) and 70 GeV (HERA). One sees that this distribution is almost independent of W. On the other hand, as expected, the distribution exhibits a kind of power-law fall-off, e.g., $t^{-5.5}$ around 3 GeV^2 and $t^{-6.5}$ around 9 GeV^2. Actually, we have found that the very simple form

$$F(t) = \frac{A}{t^5(1 + Bt + Ct^2)}, \quad (36)$$

with $A = 94.5$ nb GeV^8, $B = -0.113$ GeV^{-2}, $C = 0.043$ GeV^{-4}, provides an excellent fit of our results from $t = 2$ GeV^2 up to values of t as large as 15 GeV^2. It may be useful in a generator program for a simulation of the process. Integrating this form between 2 GeV^2 and 10 GeV^2 yields a cross section of about 1.5 nb.

Also shown for comparison in the same figure is the fit of low-t experimental data provided by the ZEUS Collaboration [20]. The observed distribution has an exponential fall-off of $\sim \exp(-bt)$ with $b \sim 7.3$ GeV^{-2} for $\langle W \rangle = 70$ GeV, in agreement with the expectation of a diffractive character of the process in that range.

The comparison in Fig. 2 is indeed encouraging for the diquark model, since a direct extrapolation from our results towards lower values of t nicely compare in magnitude with the above-mentioned data. Let us recall here that according to the well-known asymptotic constituent

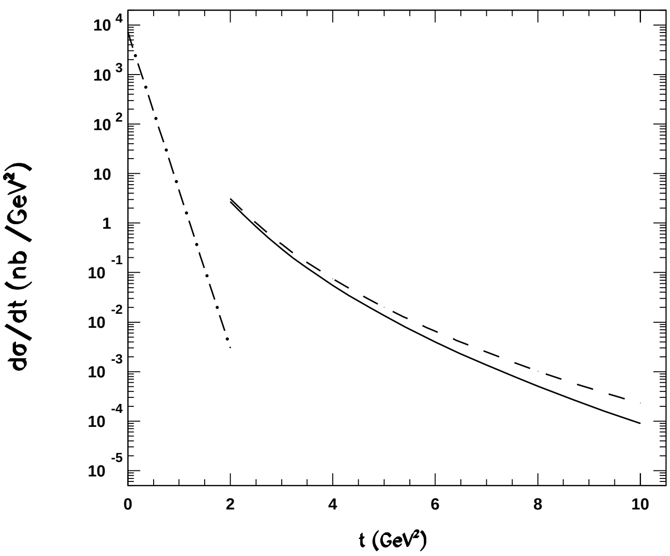

Fig. 2. Diquark-model predictions for $\mathrm{d}\sigma/\mathrm{d}t$ vs t; solid line: $W = 5$ GeV; dashed line: $W = 70$ GeV. Dash-dotted line: fit of low-t data at $< W >= 70$ GeV [20]

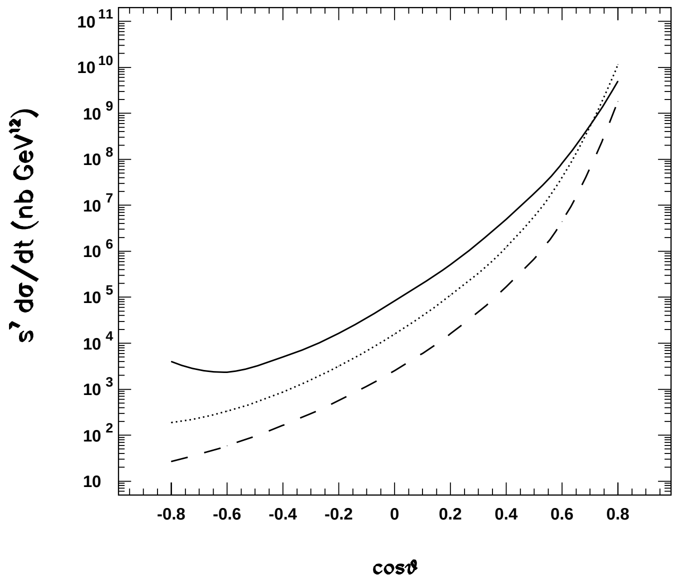

Fig. 3. Diquark-model predictions for $s^7\mathrm{d}\sigma/\mathrm{d}t$ vs $\cos\theta_{cm}$; solid line: $W = 5$ GeV; dotted line: $W = 20$ GeV; dashed line: $W = 70$ GeV

counting rule, one expects a change of the observed exponential fall-off of the t distribution at low t into a power one as t is increasing. The diquark model is just supposed to account for that transition. One may thus consider the diquark model as a good candidate in describing future data at larger t, most probably with more refined wave functions and diquark form factors. Thus a link is established, at least for that kind of process, between diffractive physics that holds at low t and the semiperturbative approach of hard exclusive processes that one expects to hold at very large t (the proton recovers there its three-quark structure).

Fig. 3 shows $s^7\mathrm{d}\sigma/\mathrm{d}t$ as a function of $\cos(\theta)$ where $s = W^2$. In the pure three-quark picture of the proton, that distribution is predicted to be independent of s at large s, provided that α_s, the strong coupling constant, is taken as a constant. Obviously, that scaling law does not hold here. Essentially, this is because we are using an evolutionary picture of the diquark structure where the α_s are expressed in running forms. It can be easily checked that one recovers the expected scaling law, if one takes both the α_s and the factor χ in diquark form factors as constants. However, numerical computations show that this works better for $W \geq 10$ GeV. In this respect, $W = 5$ GeV is not an asymptotic value.

Such deviations of the diquark model predictions from the asymptotic scaling law have been obtained also by Kroll and coworkers in their recent calculation on photoproduction of K and $K^\star$ mesons off proton; they too use the diquark model, with a similar parametrization [21]. One should be aware of the fact that in Fig. 3, very different values of t are probed for a given $\cos\theta$, depending on the value of energy W. For example, at $\cos\theta = 0.6$, the values of t corresponding to $W = 5,\ 20$, and 70 GeV are, respectively, $t = 5$ GeV^2, $t = 80$ GeV^2, and $t = 800$ GeV^2. In these various ranges, the fall-off of the cross section is very different. As has already been mentioned, it is $\sim t^{-5} \to t^{-7}$ between 2 and 15 GeV^2, but it is faster, like $t^{-8.5}$, at higher t. So, on one hand, the slopes are different for the curves corresponding to $W = 5$ GeV and, say, $W = 20$ GeV, but on the other hand, because of the multiplicative factor s^7, the yields for these two energies may be equal. This explains the intersection point of the two curves at about $\cos\theta = 0.6$. It should also be noticed that at large transfer values, the contribution of the real parts of the amplitudes may compete with those of the imaginary parts, so that our results may be only qualitative there. Anyway, it is clear that counting rates are very small in that kinematical range.

In our numerical calculations, we also tried two other parametrizations of the quark–diquark structure of the proton. The first one is that of Kroll, et al. in [22]. We found that the corresponding contribution of imaginary parts of amplitudes alone yields a t distribution that is larger than that of Fig. 2 by more than one order of magnitude; this seems to rule out that parametrization since the corresponding rates look too high, especially at low t. The second parametrization has a quark–diquark wave function derived from the asymmetrical three-quark proton wave function obtained by King and Sachrajda from QCD sum rules [23]. It is described in [16]. This time, the contribution of the imaginary parts of amplitudes is much less than that obtained from the asymptotic wave function. On the other hand, real parts seem now to contribute much more. However, as said before, the latter are difficult to evaluate properly because of instabilities in numerical computations. Thus we cannot draw any conclusion at the present time about that parametrization. This shows at least that the study of Φ photoproduction at intermediate t would allow one to discriminate between various models. At this point, one may wonder whether our calculation is

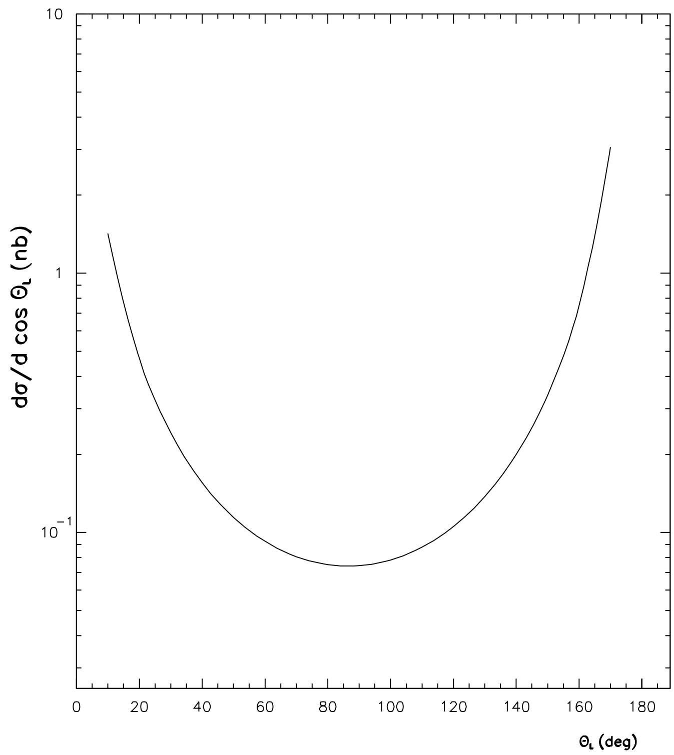

Fig. 4. Angular distribution of Φ meson (relative to the direction of the ingoing electron) in $e+p \to e+p+\Phi$ at HERA

sensitive or not to the choice of the Φ wave function. As in our numerical computation, replacing the CZ wave function by the asymptotic one $\sim 6y(1-y)$ increases the rates by about 40%. Let us also notice that according to a recent analysis by P. Ball and V.M. Braun [24], the Φ wave function is found to have a shape very close to the asymptotic one when mass corrections are discarded, as it should be done here for the coherence of our model.

For completeness, we present in Fig. 4 the angular distribution of a produced Φ meson in the full electroproduction process $e+p \to e+p+\Phi$ at HERA ($\sqrt{s} = 300$ GeV) in an untagged electron mode with photon virtuality Q^2 less than a detector limit of 4 GeV^2, so that an equivalent photon approximation can be safely applied. Here, θ_{L} is the Φ emission angle in the laboratory frame, relative to the direction of the ingoing electron. Values of the transfer t at the proton vertex have been restricted to the interval 2 GeV^2 $\leq t \leq 15$ GeV^2, but the precise value of the large upper limit is unimportant because of the rapid fall-off of the cross section at large t.

Unfortunately, exclusive photoproduction processes are as yet largely unexplored at large transfers, though they undoubtedly possess in that range a physics potential. We hope very much to dispose in the near future of new data from CEBAF or HERA, at least at intermediate transfers.

It remains now to compare the results here presented with predictions corresponding to the conventional three-quark structure of the proton. Indeed, we are just beginning a new calculation of the process on the basis of that model.

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